Correction of Exploring Properties of Cayley Graphs of $\mathbb{Z}$ with Infinite Generating Sets

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April 18, 2018

We correct the proof of Theorem 6 in [1], using all the same notations and references given in [1].

**Theorem 6.** Let $f : (\mathbb{Z}, d_2) \rightarrow (\mathbb{Z}, d_3)$ be a map defined with

$$f(a) = \sum_{i=0}^{\infty} \varepsilon_i 3^i,$$

where $a = \sum_{i=0}^{\infty} \varepsilon_i 2^i \in \mathbb{Z}$, satisfying the conditions of [14, Theorem 3].

Then $f$ is not a quasi-isometry.

**Proof.** Assume that $f$, as defined above, is a quasi-isometry. Then there exist constants $k \geq 1$ and $c \geq 0$, such that

$$\frac{1}{k} d_2(a, b) - c \leq d_3(f(a), f(b)) \leq kd_2(a, b) + c.$$

Choose $n \in \mathbb{N}$ such that $2n > 2k + c$. Then take $a = 1 + 2^2 + \cdots + 2^{2n-2}$ and $b = -2a$. By Nathanson’s length formula for $C_2$, it follows that

$$d_2(a, b) = l_2(a - b) = l_2(2^{2n} - 1) = 2.$$

In addition, by Nathanson’s length formula for $C_3$, it follows that

$$d_3(f(a), f(b)) = l_3(f(a) - f(b)) = l_3(1 + 3 + 3^2 + \cdots + 3^{2n-1}) = 2n.$$

In this case,

$$d_3(f(a), f(b)) = 2n > 2k + c = kd_2(a, b) + c,$$

which contradicts the assumption that $f$ is a quasi-isometry.

**References**