Problem Section

Problem 1: There are twelve silver coins with identical physical appearance. However there is one counterfeit coin which can be identified by its weight only, all other silver coins having the same weight. How can one identify the counterfeit coin using only three weighings on a weighing balance?

Problem 2: Suppose $f : \mathbb{R}^+ \to \mathbb{R}^+$ is continuous and

$$\int_0^\infty f(x)\,dx < \infty.$$

For any fixed $\alpha > 0$, show that

$$\lim_{n \to \infty} \frac{1}{n^\alpha} \int_0^n x^\alpha f(x)\,dx = 0.$$

Send your complete solutions to the editor.